Published by Institute of Physics Publishing for SISSA

Received: *April 3, 2007*
Accepted: *May 21, 2007*
Published: *May 29, 2007*

Determination of the neutrino mass hierarchy in the regime of small matter effect

Thomas Schwetz

CERN, Physics Department, Theory Division,
CH-1211 Geneva 23, Switzerland
E-mail: schwetz@cern.ch

Abstract: We point out a synergy between T-conjugated oscillation channels in the determination of the neutrino mass hierarchy with oscillation experiments with relatively short baselines ($L \lesssim 700$ km), where the matter effect is small. If information from all four oscillation channels $\nu_\mu \to \nu_e$, $\bar{\nu}_\mu \to \bar{\nu}_e$, $\nu_e \to \nu_\mu$ and $\bar{\nu}_e \to \bar{\nu}_\mu$ is available, a matter effect of few percent suffices to break the sign-degeneracy and allows to determine the neutrino mass hierarchy. The effect is discussed by analytical considerations of the relevant oscillation probabilities, and illustrated with numerical simulations of realistic experimental setups. Possible configurations where this method could be applied are the combination of a super beam experiment with a beta beam or a neutrino factory, or a (low energy) neutrino factory using a detector with muon and electron charge identification.

Keywords: Solar and Atmospheric Neutrinos, Neutrino Physics.

Contents

1. Introduction

The determination of the neutrino mass hierarchy is one of the most interesting open issues in neutrino physics. Present data allow for the two possibilities normal hierarchy (NH) and inverted hierarchy (IH), which are conventionally parametrized by the sign of the difference of the mass-squares of the first and third neutrino mass eigenstates: $\Delta m^2_{31} > 0$ for NH and $\Delta m^2_{31} < 0$ for IH. Identifying which of these two possibilities is realized in nature is of great importance for our understanding of the neutrino mass mechanism, the relation of neutrinos to the charged fermions, and the problem of flavour in general.

Nevertheless, the determination of the sign of Δm^2_{31} turns out to be experimentally challenging. The most promising way seems to be to explore the matter effect in neutrino oscillations [1 − 3]. This can be done with long-baseline experiments [4], atmospheric neutrinos [5], or supernova neutrinos [6]. Alternative methods to determine the hierarchy, not based on the matter effect, have been proposed [7, 8], but they turn out to be extremely challenging, if not impossible in practice.

The usual strategy to determine the mass hierarchy in long-baseline experiments is to consider configurations where the matter effect is *strong*, with baselines as long as possible, ideally several 1000 km (see, e.g., ref. [9] for a recent work), and neutrino energies close to (or above) the resonance energy [3], which for typical densities of earth matter and $|\Delta m^2_{31}| \simeq 2.5 \times 10^{-3}\,\mathrm{eV}^2$ is around 10 GeV. The basic idea is to observe the effect of the resonance, which occurs for neutrinos in case of NH and anti-neutrinos for IH. Hence, the discrimination of the two hierarchies is based on the difference of the matter effect in CP-conjugated oscillation channels, e.g., $\nu_e \to \nu_\mu$ and $\bar\nu_e \to \bar\nu_\mu$ oscillations. The ultimate setup to explore this effect is certainly a neutrino factory with one or two baselines of several thousand km, see ref. [10] for a recent study. Other options are intense super beam experiments with a detector at a baseline $L \gtrsim 1000$ km [11, 12], or very long-baseline high gamma beta beam experiments [13].

In this work we will consider a different strategy, and discuss the possibility to determine the mass hierarchy with experiments in the regime of *small* matter effect. We will consider experiments with baselines in the range 100 km $\lesssim L \lesssim$ 800 km and in the energy range of $300\,\text{MeV} \lesssim E \lesssim$ few GeV. In this regime the size of the matter effect is typically a few percent. We point out that if information from all four CP and T-conjugate oscillation channels $\nu_\mu \to \nu_e$, $\bar\nu_\mu \to \bar\nu_e$, $\nu_e \to \nu_\mu$, and $\bar\nu_e \to \bar\nu_\mu$, is available such small effects can be explored efficiently in order to determine the neutrino mass hierarchy. The basic observation is that the so-called sign-degenerate solution [14, 15], which prevents the determination of the mass hierarchy, moves in opposite directions in the plane of θ_{13} and $\delta_{\rm CP}$ for T-conjugate channels. This effect has been observed in ref. [16] in a simulation of CERN based beta beam and super beam experiments. Here we discuss the underlying physics and point out the general principle without specializing to a specific experimental configuration. This method could be applied for example for the combination of generic beta beam ($\overset{(-)}{\nu}_e \to \overset{(-)}{\nu}_\mu$) and super beam ($\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e$) experiments. Furthermore, a low energy neutrino factory [17] with a detector (or detectors) capable of muon and electron charge identification would offer a place to apply this method. If charge identification for electrons is not possible, the neutrino factory (providing the $\overset{(-)}{\nu}_e \to \overset{(-)}{\nu}_\mu$ information) could be combined with a $\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e$ super beam experiment [18].

Let us stress that we concentrate on experiments operating close to the first oscillation maximum. Hence, the following considerations do not necessarily apply to super beam experiments with neutrino energies $E \sim$ GeV and a detector at large baselines $L \gtrsim 1000$ km, far beyond the first oscillation maximum [11, 12]. Apart from the fact that in this case the matter effect cannot be considered to be small, also the rich information from the energy spectrum at higher oscillation maxima can lead to a rather different behaviour of the sign-degenerate solution than discussed here. Also, our discussion does not directly apply to a standard neutrino factory with tens of GeV neutrino energies and baselines between 3000 to 7000 km. Such an experiment works below the first maximum and is strongly affected by the matter effect. In this context the $\overset{(-)}{\nu}_e \to \overset{(-)}{\nu}_\mu$ and $\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e$ channels are called "golden" and "platinum" channels, respectively, and this combination has been investigated recently in ref. [10].

The outline of the remainder of the paper is as follows. In section 2 we discuss the proposed hierarchy determination by considerations of the relevant oscillation probabilities. We discuss the resolution of the sign-degenerate solution by using analytical expressions for the probabilities and confirm the results by numerical calculations. In section 3 the method is illustrated with simulations of realistic experimental configurations. As examples we combine the $\mu \to e$ super beam experiments SPL, T2HK, and NOνA with a $\gamma = 100$ beta beam ($e \to \mu$), all operating in the regime of relatively small matter effect. A summary is presented in section 4.

2. Resolving the $\mathrm{sgn}(\Delta m^2_{31})$-degeneracy with T-conjugated oscillation channels

In this section we discuss how the sign-degeneracy can be resolved with CP and T-conjugated channels by considering the location of the degenerate solution in the plane of θ_{13} and $\delta_{\rm CP}$. Similar considerations can be found for example in refs. [14, 15, 19, 20]. Let us depart from an expression for the appearance oscillation probability in the $\nu_e - \nu_\mu$ sector, expanded to second order in the small parameters $s_{13} \equiv \sin\theta_{13}$ and $\alpha \equiv \Delta m^2_{21}/|\Delta m^2_{31}|$ valid for constant matter density [21 – 23]:

$$\begin{aligned} P_{\rm app} \approx\ & 4\, s_{13}^2\, s_{23}^2 \frac{\sin^2\Delta(1-ahA)}{(1-ahA)^2} + \alpha^2 \sin 2\theta_{12}\, c_{23}^2 \frac{\sin^2 A\Delta}{A^2} \\ & + 2h\,\alpha\, s_{13}\, \sin 2\theta_{12}\, \sin 2\theta_{23} \cos(h\Delta - at\delta_{\rm CP}) \frac{\sin\Delta A}{A} \frac{\sin\Delta(1-ahA)}{1-ahA}\,, \end{aligned} \tag{2.1}$$

with the definitions

$$\Delta \equiv \frac{|\Delta m^2_{31}| L}{4E}\,, \quad A \equiv \left|\frac{2EV}{\Delta m^2_{31}}\right|\,, \tag{2.2}$$

where L is the baseline, E is the neutrino energy, and V is the effective matter potential [1]. The signs a, t, h describe the effects of CP-conjugation, T-conjugation, and the neutrino mass hierarchy, respectively:

$$a = \begin{cases} +1 & \text{for } \nu \\ -1 & \text{for } \bar\nu \end{cases}, \qquad t = \begin{cases} +1 & \text{for } e \to \mu \\ -1 & \text{for } \mu \to e \end{cases}, \qquad h = \mathrm{sgn}(\Delta m^2_{31})\,. \tag{2.3}$$

The matter effect enters via the parameter A. It is clear from eq. (2.1) that in the case of large matter effect $A \gtrsim 1$ the terms $(1 - ahA)$ depend strongly on the type of the mass hierarchy, and for $ah = 1$ (neutrinos and NH, or anti-neutrinos and IH) a resonance is encountered for $A = 1$ [3].

In the following we will focus on a different situation, namely the regime of small matter effect $A \ll 1$. Numerically one finds for a matter density of 3 g/cm^3

$$A \simeq 0.09 \left(\frac{E}{\rm GeV}\right) \left(\frac{|\Delta m^2_{31}|}{2.5 \times 10^{-3}\ {\rm eV}^2}\right)^{-1}\,. \tag{2.4}$$

Furthermore, we concentrate on experiments operating at the first oscillation maximum, which is characterized by $\Delta \simeq \pi/2$, or

$$E \simeq 0.2\,{\rm GeV} \left(\frac{L}{100\,{\rm km}}\right) \left(\frac{|\Delta m^2_{31}|}{2.5 \times 10^{-3}\ {\rm eV}^2}\right)\,. \tag{2.5}$$

Eqs. (2.4) and (2.5) imply that for experiments at baselines of 130 km, 295 km, 730 km the matter effect is of order 2%, 5%, 13%, respectively.[1] Keeping this in mind it makes sense

[1] A discussion of the issue in which regions of L and E the matter effect is important can be found for example in ref. [23].

JHEP05(2007)093

to expand the probability eq. (2.1) also in the small quantity A. To simplify the following equations we set $\theta_{23} = \pi/4$ and use the abbreviation

$$\tilde{\alpha} \equiv \sin 2\theta_{12} \, \frac{\Delta m^2_{21} L}{4E} \,. \tag{2.6}$$

Then eq. (2.1) becomes to first order in A

$$\begin{aligned} P_{\rm app} \approx\, & 2s_{13}^2 \sin^2\Delta + \frac{1}{2}\,\tilde{\alpha}^2 + 2h\,\tilde{\alpha}\, s_{13}\, \sin\Delta \cos(h\Delta - at\delta_{\rm CP}) \\ & + 2a\, s_{13} A\,(\sin\Delta - \Delta\cos\Delta)\,[2h\, s_{13}\sin\Delta + \tilde{\alpha}\cos(h\Delta - at\delta_{\rm CP})]\ , \end{aligned} \tag{2.7}$$

where the first line is just the vacuum probability and the second line corresponds to the leading order matter effect correction.

The reason why it is difficult to determine the neutrino mass hierarchy is a parameter degeneracy with s_{13} and $\delta_{\rm CP}$ [14, 15], i.e., for given s_{13}, $\delta_{\rm CP}$, and $\mathrm{sgn}(\Delta m^2_{31})$ in many situations the same probability can be obtained for the opposite sign of Δm^2_{31} and different values s'_{13} and $\delta'_{\rm CP}$:

$$P_{\rm app}(a,t;h,s_{13},\delta_{\rm CP}) = P_{\rm app}(a,t;-h,s'_{13},\delta'_{\rm CP})\,. \tag{2.8}$$

Assuming a given oscillation channel t and neutrino and anti-neutrino data ($a = \pm 1$) this is a system of two equations for the variables s'_{13} and $\delta'_{\rm CP}$. If a solution to this system exists the mass hierarchy cannot be determined. For example, in the case of vacuum oscillations it follows immediately from eq. (2.7) that the condition (2.8) can be fulfilled for $s'_{13} = s_{13}$ and $\delta'_{\rm CP} = \pi - \delta_{\rm CP}$ [14]:

$$P^{\rm vac}_{\rm app}(a,t;h,s_{13},\delta_{\rm CP}) = P^{\rm vac}_{\rm app}(a,t;-h,s_{13},\pi-\delta_{\rm CP})\,, \tag{2.9}$$

where $P^{\rm vac}_{\rm app}$ is given by the first line of eq. (2.7), and eq. (2.9) holds independently of a, t and the neutrino energy E.

To include the leading order matter effect correction we introduce small deviations from this vacuum solution:

$$s'_{13} = s_{13}(1+\epsilon_s)\,, \qquad \delta'_{\rm CP} = \pi - \delta_{\rm CP} + \epsilon_\delta\,, \tag{2.10}$$

with $\epsilon_s, \epsilon_\delta \ll 1$. Using eqs. (2.7) and (2.10) in eq. (2.8) and expanding to first order in $\epsilon_s, \epsilon_\delta$, and A yields the condition

$$[\epsilon_s - 2ahA(1-\Delta\cot\Delta)]\,[2h\, s_{13}\sin\Delta + \tilde{\alpha}\cos(h\Delta - at\delta_{\rm CP})] = at\epsilon_\delta\tilde{\alpha}\sin(h\Delta - at\delta_{\rm CP})\,. \tag{2.11}$$

For a given "true" hierarchy h, a fixed oscillation channel t, and $a = \pm 1$ this is a linear system of two equations for ϵ_s and ϵ_δ, which in general has a unique solution. Hence, for neutrino plus anti-neutrino data in one oscillation channel the leading order matter effect cannot break the degeneracy. This is the reason why experiments at relatively small baselines ($L \lesssim 700$ km) have very poor sensitivity to the mass hierarchy.[2] In order to resolve

[2]Eq. (2.11) can be fulfilled exactly only for one energy. Hence, in principle spectral information can be used to resolve the degeneracy. Note, however, that this is an effect at third order in the small quantities $\epsilon_s, \epsilon_\delta, A, s_{13}, \tilde{\alpha}$. Hence, it is difficult to obtain enough statistics to explore spectral information.

JHEP05(2007)093

the degeneracy one has to enter the regime of *large* matter effect, where the non-linear character of eq. (2.1) becomes relevant and prevents a solution of the two conditions (2.8) for fixed t and $a \pm 1$.

However, the immediate conclusion from eq. (2.11) is that if all CP and T-conjugate channels are available one obtains four independent relations (corresponding to $a = \pm 1$ and $t = \pm 1$) for the two variables ϵ_s and ϵ_δ. Obviously, in general such a system has no solution and hence the degeneracy is broken. To illustrate this explicitly let us consider for simplicity the case $\Delta = \pi/2$, i.e., experiments exactly at the first oscillation maximum. Then eq. (2.11) simplifies to

$$(\epsilon_s - 2ahA)(2s_{13} + at\,\tilde{\alpha}\sin\delta_{\rm CP}) = at\,\epsilon_\delta\,\tilde{\alpha}\cos\delta_{\rm CP}\,. \tag{2.12}$$

For $a = \pm 1$ and given t this system of equations has the solution

$$\begin{aligned}
\epsilon_s &= ht\,A\,\frac{\tilde{\alpha}}{s_{13}}\,\sin\delta_{\rm CP}\,,\\
\epsilon_\delta &= ht\,\frac{A}{\cos\delta_{\rm CP}}\,\left(\frac{\tilde{\alpha}}{s_{13}}\,\sin^2\delta_{\rm CP} - 4\frac{s_{13}}{\tilde{\alpha}}\right)\,.
\end{aligned} \tag{2.13}$$

The crucial observation from these expressions is that the signs of both, ϵ_s and ϵ_δ, depend on the oscillation channel t. Hence, with increasing matter effect A the location of the solution with the wrong hierarchy in the $\theta_{13} - \delta_{\rm CP}$ plane moves in opposite directions for $\mu \to e$ and $e \to \mu$ transitions.

We have verified this behaviour by numerical calculations of the oscillation probability for constant matter (without any expansion in small quantities). Based on such calculations we graphically solve the system of equations (2.8) in figure 1. The appearance probability is calculated for NH and the parameters $\sin^2 2\theta_{13} = 0.02$ and $\delta_{\rm CP} = 36^\circ$ (marked as a star). Then the curves in figure 1 correspond to the set of values $\sin^2 2\theta'_{13}$ and $\delta'_{\rm CP}$ leading to the same probability but for IH. In each panel there are four curves, corresponding to the four combinations of neutrinos/anti-neutrinos and $(e \to \mu)/(\mu \to e)$. The dots show the location of the degenerate solutions, where the curves for neutrinos and anti-neutrinos for a given channel cross. The second crossing close to the original parameter values correspond to the so-called intrinsic degenerate solution [24, 15] (with the wrong hierarchy). For experiments at the first oscillation maximum this degeneracy is resolved quite efficiently by spectral information, and hence, in many cases it does not appear as a viable solution (see, e.g., ref. [25] for an explicit discussion in the case of the T2HK experiment). Therefore we will neglect it in the present discussion and focus on the solutions marked with dots in figure 1.

In the upper-left panel of figure 1 we consider a (hypothetical) experiment at a very short baseline $L = 10$ km and an energy of 0.02 GeV, where the matter effect is negligible, see eq. (2.4). One finds that all four curves meet in the point corresponding to eq. (2.9) with $\delta'_{\rm CP} = \pi - \delta_{\rm CP}$, which makes the determination of the hierarchy impossible even if all CP and T-conjugated channels are available. By increasing the baseline (and simultaneously choosing the energy to stay in the first oscillation maximum) one observes from the plots that the degenerate solutions for the $e \to \mu$ and $\mu \to e$ channels separate and move in opposite directions, in agreement with eq. (2.13).

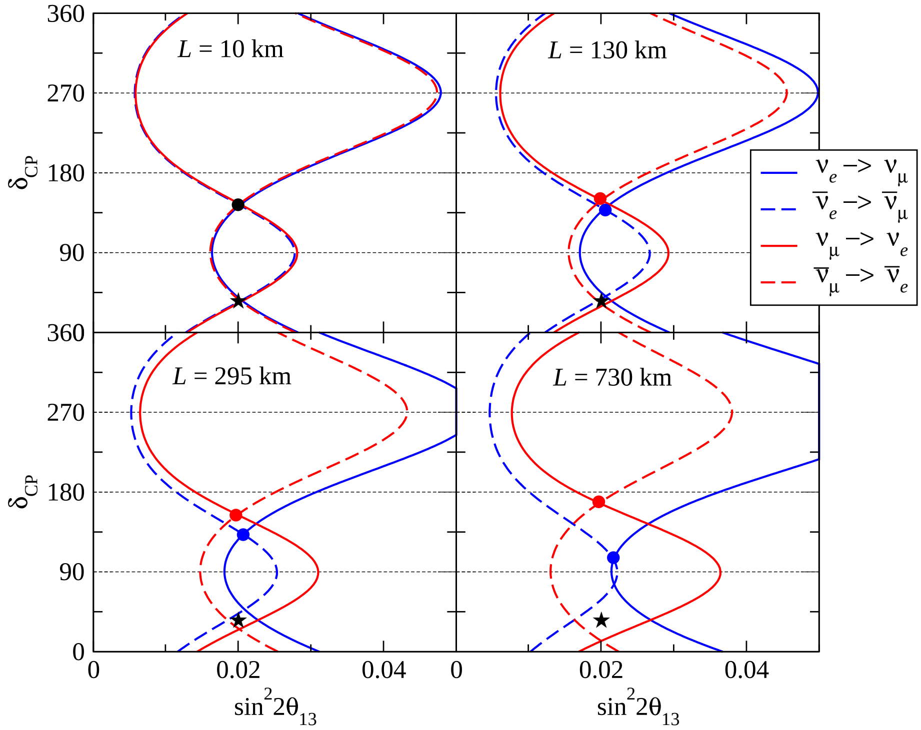

Figure 1: Location of the sign-degenerate solutions for different baselines. The plots show a graphical representation of the equations $P_{\rm app}(a,t;{\rm NH},s_{13},\delta_{\rm CP}) = P_{\rm app}(a,t;{\rm IH},s'_{13},\delta'_{\rm CP})$ for $a = \pm 1$ (neutrinos/anti-neutrinos) and $t = \pm 1$ $((e \to \mu)/(\mu \to e))$. The star indicates the assumed values for $\sin^2 2\theta_{13}$ and $\delta_{\rm CP}$ for the NH, whereas the axes correspond to the primed parameters for the wrong hierarchy. The dots show the location of the degenerate solutions, where the curves for neutrinos and anti-neutrinos for fixed t cross. The four panels correspond to different baselines, and the neutrino energy in each panel is determined by assuming the first oscillation maximum and $|\Delta m^2_{31}| = 2.5 \times 10^{-3}\,{\rm eV}^2$, see eq. (2.5).

Exp.	Ref.	L [km]	$\langle E \rangle$ [GeV]	Detector	Time [yr]	Beam	$\sigma_{\rm sys}$
BB	[16]	130	0.4	500 kt WC	$4\nu + 4\bar\nu$	$2.2\,(5.8) \times 10^{18}$	2%
SPL	[16]	130	0.3	500 kt WC	$2\nu + 8\bar\nu$	4 MW	2%
T2HK	[27]	295	0.8	500 kt WC	$4\nu + 4\bar\nu$	4 MW	5%
NOνA	[28]	812	2.0	25 kt TASD	$3\nu + 3\bar\nu$	1.12 MW	5%

Table 1: Main parameters of the simulated setups [26]. For BB the "beam" column corresponds to the number of useful ^{18}Ne (^{6}He) decays per year, whereas for the super beams the beam power is given. The systematical error $\sigma_{\rm sys}$ corresponds to the uncertainty on the signal and background rates, uncorrelated between signal, background, neutrinos, and anti-neutrinos.

3. Numerical simulations

In this section we show by numerical simulations of realistic experimental configurations how one can benefit from the synergy of T-conjugated oscillation channels. We consider the three super beam experiments SPL, T2HK, and NOνA, providing the $\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e$ information, as well as a $\gamma = 100$ beta beam (BB) experiment operating in the $\overset{(-)}{\nu}_e \to \overset{(-)}{\nu}_\mu$ channels. The simulation is performed with the GLoBES software [26], and all setups correspond to the pre-defined configurations provided by GLoBES 3.0. The most relevant parameters for each experiment are given in table 1. BB and SPL are CERN based experiments, using the 500 kt water Čerenkov (WC) detector MEMPHYS at Frejus, at a distance of 130 km, details can be found in ref. [16]. With such a short baseline and low energies the matter effect is very small. T2HK is the second phase of the T2K experiment [27] in Japan, based on a 4 MW upgrade of the beam and the 500 kt HyperKamiokande detector. Further details of the simulation can be found also in ref. [25]. In this experiment the matter effect is somewhat larger than for the CERN-MEMPHYS configuration, but still too small to explore the neutrino mass hierarchy. Finally, we consider the Fermilab based NOνA experiment [28] with a 25 kt totally active scintillator detector (TASD) at a baseline of 812 km, where the matter effect starts to be important. Note however, that here we consider an initial stage setup for NOνA, with significantly less statistics than the other configurations.

To simulate the data we take the following values for the oscillation parameters: $\Delta m^2_{21} = 7.9 \times 10^{-5}\,\mathrm{eV}^2$, $\sin^2\theta_{12} = 0.3$, $\Delta m^2_{31} = +2.4 \times 10^{-3}\,\mathrm{eV}^2$, $\sin^2\theta_{23} = 0.5$, and we assume an external uncertainty of 4% on the solar parameters, and 5% on the matter density uncertainty along the baseline of each experiment. Note that each experiment includes neutrino and anti-neutrino data, and appearance and disappearance channels are used in the analysis.

In figure 2 we show an example of the sign-degenerate solutions in the $\sin^2 2\theta_{13} - \delta_{\mathrm{CP}}$ plane for the four experiments of table 1. This plot confirms the behaviour discussed in the previous section on probability level by performing an actual fit to simulated data: One can see that the best fit point with the wrong hierarchy moves in opposite directions for BB ($e \to \mu$) and the super beam experiments ($\mu \to e$), relative to the vacuum solution at $\delta_{\mathrm{CP}} = \pi - \delta_{\mathrm{CP}}^{\mathrm{true}}$, which is indicated by the dashed line. Furthermore, the dislocation of the degeneracy for the super beam experiments from the vacuum value of δ_{CP} is proportional to the baseline, i.e., increasing in the order SPL, T2HK, NOνA. This behaviour shows that the combination of $\mu \to e$ and $e \to \mu$ experiments offers a promising synergy in resolving the sign-degeneracy. Note that that in all cases only one solution appears with the wrong hierarchy, which justifies to neglect the intrinsic degeneracy in the discussion of the previous section.

Let us stress that in each case a detailed quantitative study is necessary in order to fully assess the potential of this method. The final ability to disfavour the degenerate solution depends on many details which affect the size of the allowed regions for the individual experiments. For example, the relatively large allowed region for NOνA seen in figure 2, which is a consequence of the much smaller statistics compared to the other setups, will certainly limit the sensitivity, tough the best fit point of the degeneracy for NOνA clearly follows the trend discussed above. For the SPL/BB combination the effect discussed here has apparently not been found in refs. [20, 29]. It is clear from figure 2 that for SPL+BB a slight enlargement of the allowed regions would be enough to corrupt the ability to resolve the sign-degeneracy. The size of the allowed regions depends rather sensitively on the details

JHEP05(2007)093

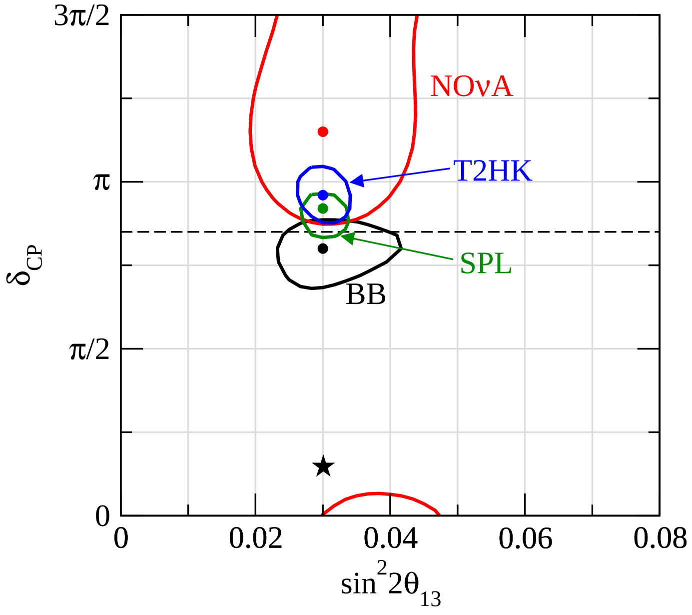

Figure 2: Allowed regions at 90% CL in the $\sin^2 2\theta_{13} - \delta_{\rm CP}$ plane with the wrong hierarchy for BB, SPL, T2HK, and NOνA. Data is simulated for NH and $\sin^2 2\theta_{13}^{\rm true} = 0.03$, $\delta_{\rm CP}^{\rm true} = 0.15\pi$, marked with a star. The dots show the best fit point with IH hierarchy. The horizontal dashed line indicates the value $\delta_{\rm CP} = \pi - \delta_{\rm CP}^{\rm true}$, corresponding to the location of the degenerate solution in vacuum, see eq. (2.9). For the BB external accuracies on $|\Delta m^2_{31}|$ and θ_{23} of 5% and 10% have been assumed, respectively.

of the analysis (see e.g., figure 6 of ref. [16] for the case of SPL). A possible reason for the differences with refs. [20, 29] might be for example the assumptions on systematics (see table 1), or the inclusion of spectral information for appearance as well as disappearance channels, which has quite a significant impact on the size of the allowed regions [16].

In figure 2 we have assumed a "true" NH. If the true hierarchy is inverted the degenerate solutions for the $\mu \to e$ and $e \to \mu$ channels move into the opposite directions as in the case of a true NH. This follows from eq. (2.13), where h corresponds to the true sign of Δm^2_{31}. Of course, the complementarity between the T-conjugated channels remains independently of the true hierarchy.

Let us comment on the widely discussed strategy of using information from two $\mu \to e$ experiments at similar L/E but different baselines, see, e.g., ref. [30]. Such a situation is realized by the combination of T2K and NOνA [31] or by placing a second detector in the NOνA beam-line at a suitable off-axis angle, as proposed in ref. [32]. It is clear from the preceding discussion and from figure 2 that the synergy from such a combination is less effective than using $e \to \mu$ information. The reason is that the degeneracies for two $\mu \to e$ experiments move in the *same* direction in the $\sin^2 2\theta_{13} - \delta_{\rm CP}$ space, only the size of the dislocation is different due to the different baselines.

In figure 3 we show how the combination of the super beam experiments SPL, T2HK, and NOνA with the $\gamma = 100$ beta beam significantly enhances the sensitivity to the mass hierarchy due to the $(\mu \to e)/(e \to \mu)$ synergy. The dashed curves show the sensitivities of

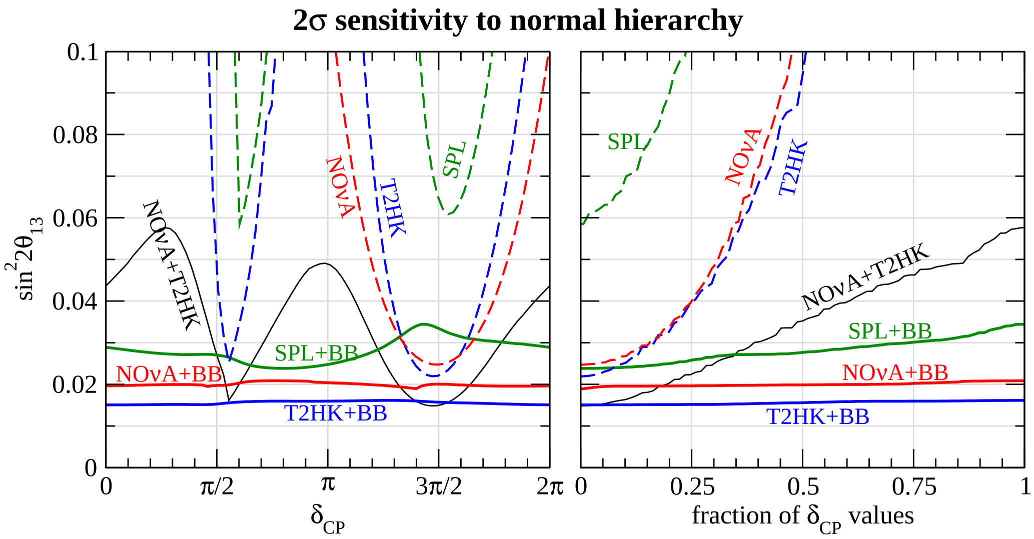

Figure 3: Minimal value of $\sin^2 2\theta_{13}$ for which the IH can be excluded at 2σ ($\Delta\chi^2 = 4$) if the true hierarchy is normal, as a function of $\delta_{\rm CP}$ (left) and the fraction of all possible values of $\delta_{\rm CP}$ (right). The dashed lines correspond to the super beam experiments SPL, T2HK, NOνA, whereas for the solid lines each of these super beams is combined with the beta beam. For comparison also the combination NOνA+T2HK is shown.

the super beams alone. One observes that these experiments can assess the mass hierarchy only in a certain range of $\delta_{\rm CP}$ values, and there is no sensitivity even for $\sin^2 2\theta_{13} = 0.1$ for 75% (50%) of all values of $\delta_{\rm CP}$ for SPL (T2HK, NOνA). Note that NOνA and T2HK have a rather similar sensitivity, where NOνA has the advantage of the longer baseline of 812 km, whereas the short baseline of T2HK of 295 km is compensated by the large statistics implied by the 4 MW beam and 500 kt detector. The main limitation of SPL is of course the short baseline of 130 km.

When these experiments are combined with the BB the sensitivity is significantly improved, see solid lines in figure 3. The dependence on $\delta_{\rm CP}$ practically disappears and a stable sensitivity is obtained for $\sin^2 2\theta_{13} \gtrsim 0.02 - 0.03$.[3] The effect is most remarkable for SPL+BB [16], since none of these experiments on its own has any notable sensitivity. Indeed, in the parameter range shown in figure 3 there is no sensitivity for BB alone. The sensitivity of the SPL+BB combination fully emerges from the complementarity of T-conjugated channels in the small matter effect regime. Also for T2HK and NOνA the effect is clearly visible, and again the larger baseline of NOνA is compensated by statistics in T2HK. For comparison we show also the combination NOνA+T2HK (without BB) in figure 3 (see ref. [31] for detailed discussions of the case NOνA + T2K phase I). Also in this case the sensitivity improves, however the complementarity is much less pronounced, a dependence on $\delta_{\rm CP}$ remains, and the effect is more similar to the addition of statistics than a true synergy (see also the corresponding discussion related to figure 2).

As a side remark let us mention the possibility pointed out in the second paper of

[3] In figure 3 we have assumed that the true hierarchy is normal, but we have checked that for a true IH the results are very similar.

ref. [7], that very accurate measurements of the neutrino mass-squared difference in ν_e and ν_μ disappearance experiments allow in principle to distinguish between NH and IH (even in vacuum). In the beta beam/super beam combination both disappearance probabilities P_{ee} and $P_{\mu\mu}$ are observed. However, we have checked that for the experiments considered here numerically this effect is completely negligible, and practically the full sensitivity shown in figure 3 emerges from the matter effect in the appearance channels.

4. Summary and discussion

In this letter we have considered neutrino oscillation experiments operating at the first oscillation maximum in the regime of small matter effect, i.e., at relatively short baselines of several hundred km. In such a case there is very poor sensitivity to the neutrino mass hierarchy, if only data from neutrinos and anti-neutrinos are available in a fixed oscillation channel. The reason is that the leading order correction in the small matter effect parameter A cannot break the $\mathrm{sgn}(\Delta m^2_{31})$-degeneracy. The usual strategy to resolve the degeneracy is to enter the regime of strong matter effect (by going to longer baselines and higher neutrino energies), where the non-linear dependence of A becomes important. Here we have proposed an alternative method, namely the combination of all four CP and T-conjugated oscillation channels. We have shown that the location of the sign-degenerate solution in the plane of θ_{13} and $\delta_{\rm CP}$ moves in opposite directions for the $\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e$ and $\overset{(-)}{\nu}_e \to \overset{(-)}{\nu}_\mu$ channels when the matter effect increases. This synergy allows to resolve the degeneracy even for matter effects as small as a few percent. We have discussed the method at the level of oscillation probabilities, and illustrated the effect also by simulations of representative experimental configurations.

A typical situation where our method applies is the combination of beta beam ($e \to \mu$) and super beam ($\mu \to e$) experiments. We have demonstrated that a significant synergy exists for the determination of the mass hierarchy by simulations of the SPL, T2HK, and NOνA super beams combined with a $\gamma = 100$ beta beam. Most remarkable, even for the SPL/beta beam combination, where both experiments have a baseline of only 130 km, there is sensitivity to the mass hierarchy for $\sin^2 2\theta_{13} \gtrsim 0.03$ due to the synergy of the T-conjugated channels. Another possibility to take advantage of this effect could be a low energy neutrino factory [17] operating in an L/E regime where the matter effect is not yet fully developed. The liquid argon technology, which has been considered for the detector in that reference, has excellent sensitivity for muon as well as electron detection. If in both cases the charge can be identified all four oscillation channels $\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e$ and $\overset{(-)}{\nu}_e \to \overset{(-)}{\nu}_\mu$ were available at the neutrino factory. If charge identification is not possible for electrons one could combine the neutrino factory with a super beam experiment providing the $\mu \to e$ information.

The purpose of this note is to point out the existence of a synergy of T-conjugated channels, which can significantly increase the sensitivity to the neutrino mass hierarchy. Whether this method for the mass hierarchy determination is indeed useful for a given experimental configuration, or is competitive with alternative approaches needs to be confirmed by detailed simulations and comparison studies, which is beyond the scope of this work.

JHEP05(2007)093

References

[1] L. Wolfenstein, *Neutrino oscillations in matter*, *Phys. Rev.* **D 17** (1978) 2369; *Neutrino oscillations and stellar collapse*, *Phys. Rev.* **D 20** (1979) 2634.

[2] V.D. Barger, K. Whisnant, S. Pakvasa and R.J.N. Phillips, *Matter effects on three-neutrino oscillations*, *Phys. Rev.* **D 22** (1980) 2718.

[3] S.P. Mikheev and A.Y. Smirnov, *Resonance enhancement of oscillations in matter and solar neutrino spectroscopy*, *Sov. J. Nucl. Phys.* **42** (1985) 913 [*Yad. Fiz.* **42** (1985) 1441]; *Resonant amplification of neutrino oscillations in matter and solar neutrino spectroscopy*, *Nuovo Cim.* **C9** (1986) 17.

[4] M. Freund, M. Lindner, S.T. Petcov and A. Romanino, *Testing matter effects in very long baseline neutrino oscillation experiments*, *Nucl. Phys.* **B 578** (2000) 27 [hep-ph/9912457]; V.D. Barger, S. Geer, R. Raja and K. Whisnant, *Determination of the pattern of neutrino masses at a neutrino factory*, *Phys. Lett.* **B 485** (2000) 379 [hep-ph/0004208].

[5] E.K. Akhmedov, A. Dighe, P. Lipari and A.Y. Smirnov, *Atmospheric neutrinos at Super-Kamiokande and parametric resonance in neutrino oscillations*, *Nucl. Phys.* **B 542** (1999) 3 [hep-ph/9808270];
M. Chizhov, M. Maris and S.T. Petcov, *On the oscillation length resonance in the transitions of solar and atmospheric neutrinos crossing the earth core*, hep-ph/9810501; J. Bernabeu, S. Palomares-Ruiz and S.T. Petcov, *Atmospheric neutrino oscillations, θ_{13} and neutrino mass hierarchy*, *Nucl. Phys.* **B 669** (2003) 255 [hep-ph/0305152];
D. Indumathi and M.V.N. Murthy, *A question of hierarchy: matter effects with atmospheric neutrinos and anti-neutrinos*, *Phys. Rev.* **D 71** (2005) 013001 [hep-ph/0407336];
P. Huber, M. Maltoni and T. Schwetz, *Resolving parameter degeneracies in long-baseline experiments by atmospheric neutrino data*, *Phys. Rev.* **D 71** (2005) 053006 [hep-ph/0501037];
S.T. Petcov and T. Schwetz, *Determining the neutrino mass hierarchy with atmospheric neutrinos*, *Nucl. Phys.* **B 740** (2006) 1 [hep-ph/0511277].

[6] C. Lunardini and A.Y. Smirnov, *Probing the neutrino mass hierarchy and the 13-mixing with supernovae*, *JCAP* **06** (2003) 009 [hep-ph/0302033];
A.S. Dighe, M.T. Keil and G.G. Raffelt, *Detecting the neutrino mass hierarchy with a supernova at icecube*, *JCAP* **06** (2003) 005 [hep-ph/0303210];
M. Kachelriess and R. Tomas, *Identifying the neutrino mass hierarchy with supernova neutrinos*, hep-ph/0412100;
V. Barger, P. Huber and D. Marfatia, *Supernova neutrinos can tell us the neutrino mass hierarchy independently of flux models*, *Phys. Lett.* **B 617** (2005) 167 [hep-ph/0501184].

[7] S.T. Petcov and M. Piai, *The LMA MSW solution of the solar neutrino problem, inverted neutrino mass hierarchy and reactor neutrino experiments*, *Phys. Lett.* **B 533** (2002) 94 [hep-ph/0112074];
H. Nunokawa, S.J. Parke and R. Zukanovich Funchal, *Another possible way to determine the neutrino mass hierarchy*, *Phys. Rev.* **D 72** (2005) 013009 [hep-ph/0503283];
A. de Gouvea, J. Jenkins and B. Kayser, *Neutrino mass hierarchy, vacuum oscillations and vanishing U_{e3}*, *Phys. Rev.* **D 71** (2005) 113009 [hep-ph/0503079];
A. de Gouvea and W. Winter, *What would it take to determine the neutrino mass hierarchy if θ_{13} were too small?*, *Phys. Rev.* **D 73** (2006) 033003 [hep-ph/0509359].

[8] S. Pascoli, S.T. Petcov and T. Schwetz, *The absolute neutrino mass scale, neutrino mass spectrum, majorana CP-violation and neutrinoless double-beta decay*, *Nucl. Phys.* **B 734** (2006) 24 [hep-ph/0505226];
S. Choubey and W. Rodejohann, *Neutrinoless double beta decay and future neutrino oscillation precision experiments*, *Phys. Rev.* **D 72** (2005) 033016 [hep-ph/0506102];
A. de Gouvea and J. Jenkins, *Non-oscillation probes of the neutrino mass hierarchy and vanishing* $|U_{e3}|$, hep-ph/0507021.

[9] R. Gandhi, P. Ghoshal, S. Goswami, P. Mehta and S. Uma Sankar, *Earth matter effects at very long baselines and the neutrino mass hierarchy*, *Phys. Rev.* **D 73** (2006) 053001 [hep-ph/0411252].

[10] P. Huber, M. Lindner, M. Rolinec and W. Winter, *Optimization of a neutrino factory oscillation experiment*, *Phys. Rev.* **D 74** (2006) 073003 [hep-ph/0606119].

[11] M.V. Diwan et al., *Very long baseline neutrino oscillation experiments for precise measurements of mixing parameters and CP-violating effects*, *Phys. Rev.* **D 68** (2003) 012002 [hep-ph/0303081];
V. Barger et al., *Precision physics with a wide band super neutrino beam*, *Phys. Rev.* **D 74** (2006) 073004 [hep-ph/0607177].

[12] M. Ishitsuka, T. Kajita, H. Minakata and H. Nunokawa, *Resolving neutrino mass hierarchy and CP degeneracy by two identical detectors with different baselines*, *Phys. Rev.* **D 72** (2005) 033003 [hep-ph/0504026];
T. Kajita, H. Minakata, S. Nakayama and H. Nunokawa, *Resolving eight-fold neutrino parameter degeneracy by two identical detectors with different baselines*, *Phys. Rev.* **D 75** (2007) 013006 [hep-ph/0609286];
K. Hagiwara, N. Okamura and K.-i. Senda, *Solving the neutrino parameter degeneracy by measuring the T2K off-axis beam in korea*, *Phys. Lett.* **B 637** (2006) 266 [*Erratum ibid.* **641** (2006) 486] [hep-ph/0504061].

[13] J. Burguet-Castell, D. Casper, J.J. Gomez-Cadenas, P. Hernandez and F. Sanchez, *Neutrino oscillation physics with a higher gamma beta-beam*, *Nucl. Phys.* **B 695** (2004) 217 [hep-ph/0312068];
P. Huber, M. Lindner, M. Rolinec and W. Winter, *Physics and optimization of beta-beams: From low to very high gamma*, *Phys. Rev.* **D 73** (2006) 053002 [hep-ph/0506237];
S.K. Agarwalla, S. Choubey and A. Raychaudhuri, *Neutrino mass hierarchy and* θ_{13} *with a magic baseline beta-beam experiment*, *Nucl. Phys.* **B 771** (2007) 1 [hep-ph/0610333].

[14] H. Minakata and H. Nunokawa, *Exploring neutrino mixing with low energy superbeams*, *JHEP* **10** (2001) 001 [hep-ph/0108085].

[15] V. Barger, D. Marfatia and K. Whisnant, *Breaking eight-fold degeneracies in neutrino CP-violation, mixing and mass hierarchy*, *Phys. Rev.* **D 65** (2002) 073023 [hep-ph/0112119].

[16] J.E. Campagne, M. Maltoni, M. Mezzetto and T. Schwetz, *Physics potential of the CERN-memphys neutrino oscillation project*, *JHEP* **04** (2007) 003 [hep-ph/0603172].

[17] S. Geer, O. Mena and S. Pascoli, *A low energy neutrino factory for large* θ_{13}, *Phys. Rev.* **D 75** (2007) 093001 [hep-ph/0701258].

[18] J. Burguet-Castell, M.B. Gavela, J.J. Gomez-Cadenas, P. Hernandez and O. Mena, *Superbeams plus neutrino factory: the golden path to leptonic CP-violation*, *Nucl. Phys.* **B 646** (2002) 301 [hep-ph/0207080].

JHEP05(2007)093

[19] H. Minakata, H. Nunokawa and S.J. Parke, *Parameter degeneracies in neutrino oscillation measurement of leptonic CP and T violation*, *Phys. Rev.* **D 66** (2002) 093012 [hep-ph/0208163].

[20] A. Donini, D. Meloni and S. Rigolin, *Clone flow analysis for a theory inspired neutrino experiment planning*, *JHEP* **06** (2004) 011 [hep-ph/0312072].

[21] A. Cervera et al., *Golden measurements at a neutrino factory*, *Nucl. Phys.* **B 579** (2000) 17 [*Erratum ibid.* **593** (2001) 731] [hep-ph/0002108].

[22] M. Freund, *Analytic approximations for three neutrino oscillation parameters and probabilities in matter*, *Phys. Rev.* **D 64** (2001) 053003 [hep-ph/0103300].

[23] E.K. Akhmedov, R. Johansson, M. Lindner, T. Ohlsson and T. Schwetz, *Series expansions for three-flavor neutrino oscillation probabilities in matter*, *JHEP* **04** (2004) 078 [hep-ph/0402175].

[24] J. Burguet-Castell, M.B. Gavela, J.J. Gomez-Cadenas, P. Hernandez and O. Mena, *On the measurement of leptonic CP-violation*, *Nucl. Phys.* **B 608** (2001) 301 [hep-ph/0103258].

[25] P. Huber, M. Lindner and W. Winter, *Superbeams versus neutrino factories*, *Nucl. Phys.* **B 645** (2002) 3 [hep-ph/0204352].

[26] P. Huber, M. Lindner and W. Winter, *Simulation of long-baseline neutrino oscillation experiments with globes*, *Comput. Phys. Commun.* **167** (2005) 195 [hep-ph/0407333];
P. Huber, J. Kopp, M. Lindner, M. Rolinec and W. Winter, *New features in the simulation of neutrino oscillation experiments with globes 3.0*, hep-ph/0701187.

[27] The T2K collaboration, Y. Itow et al., *The JHF-Kamioka neutrino project*, hep-ex/0106019.

[28] NOvA collaboration, D.S. Ayres et al., *Nova proposal to build a 30-kiloton off-axis detector to study neutrino oscillations in the Fermilab NuMI beamline*, hep-ex/0503053.

[29] A. Donini, E. Fernandez-Martinez, P. Migliozzi, S. Rigolin and L. Scotto Lavina, *STUdy of the eightfold degeneracy with a standard beta-beam and a super-beam facility*, *Nucl. Phys.* **B 710** (2005) 402 [hep-ph/0406132].

[30] V. Barger, D. Marfatia and K. Whisnant, *How two neutrino superbeam experiments do better than one*, *Phys. Lett.* **B 560** (2003) 75 [hep-ph/0210428].

[31] P. Huber, M. Lindner and W. Winter, *Synergies between the first-generation JHF-SK and NuMI superbeam experiments*, *Nucl. Phys.* **B 654** (2003) 3 [hep-ph/0211300];
H. Minakata, H. Nunokawa and S.J. Parke, *The complementarity of eastern and western hemisphere long-baseline neutrino oscillation experiments*, *Phys. Rev.* **D 68** (2003) 013010 [hep-ph/0301210];
O. Mena and S.J. Parke, *Untangling CP-violation and the mass hierarchy in long baseline experiments*, *Phys. Rev.* **D 70** (2004) 093011 [hep-ph/0408070];
O. Mena, H. Nunokawa and S.J. Parke, *Nova and T2K: the race for the neutrino mass hierarchy*, *Phys. Rev.* **D 75** (2007) 033002 [hep-ph/0609011].

[32] O. Mena Requejo, S. Palomares-Ruiz and S. Pascoli, *Super-nova: a long-baseline neutrino experiment with two off-axis detectors*, *Phys. Rev.* **D 72** (2005) 053002 [hep-ph/0504015]; *Determining the neutrino mass hierarchy and CP-violation in nonua with a second off-axis detector*, *Phys. Rev.* **D 73** (2006) 073007 [hep-ph/0510182].

JHEP05(2007)093